

## IDEAS FOR LECTURE SESSIONS BY RMS FACULTY

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The focus should be on pushing the topic to the 'edge' – to show how elementary ideas can yield insightful solutions to non-trivial problems as well as real-life applications.

### PRE-CALCULUS (ALGEBRA, COORDINATE GEOMETRY, ETC) & ANALYSIS

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1. Solution of cubic equations (Cardano's solution, trigonometric solution)
2. Historical discovery of complex numbers
3. Development of the number system: From  $N$  to  $Z$  to  $Q$  to  $R$  to  $C$
4. Proof by induction (elementary problems not generally seen at school level)
5. Formulas for sums of squares and cubes of  $\{1, 2, 3, \dots, n\}$ , non-inductive proofs
6. Application of factor theorem to factorizing polynomials and numbers
7. Indeterminate forms and L'Hopital's rule
8. Curve sketching, inflection points, significance of the second derivative
9. Geometrical proofs of some results on conics (ellipse, parabola)

### GEOMETRY (RESULTS NOT GENERALLY SEEN AT SCHOOL LEVEL)

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1. Nice proofs in geometry using trigonometry, vectors and/or complex numbers:
  - a. Euler line and nine-point circle theorem (vectors; complex numbers)
  - b. Ceva's and Menelaus's theorems (vectors)
  - c. Ptolemy's theorem (pure geometry; trigonometry)
  - d. Steiner-Lehmus theorem (trigonometry)
  - e. Napoleon's theorem (trigonometry; complex numbers)
  - f. Morley's theorem (trigonometry)
2. Problems of maxima and minima in geometry (non-calculus solutions)

### COMBINATORICS, PROBABILITY & NUMBER THEORY

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1. Conditional probability and Bayes theorem
2. Significance and subtleties of expected value of a random variable
3. Principle of inclusion-exclusion, simple problems
4. Interesting elementary identities among the binomial coefficients
5. Euler-Poincare theorem,  $V - E + F = 2$  (proof by induction)
6. Occurrence of Fibonacci numbers in counting, properties of Fibonacci numbers
7. Congruences, Wilson's theorem, Fermat's little theorem (proof by induction, also proof by counting)
8. Properties of Pythagorean triples
9. Recursion, e.g., Tower of Brahma
10. Euclid's GCD algorithm, proof that it gives the correct answer
11. Irrationality of some numbers