



S. Sivasankaranarayana
24-9-48 Pillai

DR. S. SIVASANKARANARAYANA PILLAI
(1901-1950)

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Preface

S. Sivasankaranarayana Pillai (1901–50) is probably best known to the mathematical community for the Waring’s Problem and Exponential Diophantine Equations. He has influenced number theory with his valuable contributions, as Pillai’s theorems, questions and conjectures. They have led the way to a considerable research. Thus it is appropriate to make Pillai’s papers easily available through the publication of his *Collected Works*, since most of his papers published in Indian Journals might be difficult to obtain.

Pillai had published his papers during the years 1927–1946. Though it seems that he had not published his works during the last four years of his lifetime, we have evidence that he obtained many results during 1947–1950. Although we have made every effort to locate all of Pillai’s papers (both published and unpublished), it is possible that we may have missed a few. For instance, we do have some references quoted in Pillai’s correspondence and published papers to some results of his that are either a part of his M.Sc / D. Sc. Thesis or in his unpublished work. Since we could not locate them, we have listed these results in a separate article, namely, *‘The results of S. S. Pillai that are not in this collection’*.

All the published papers of Pillai are distributed in the first two volumes and the third volume consists of unpublished works and letters from many mathematicians to Pillai including T. Vijayaraghavan, S. D. Chowla, S. S. Chern and K. Chandrasekharan etc. From the letters, one may see that in the initial days of research career of Pillai, he had regular letter correspondence with S. D. Chowla and in the later days, he had more letter correspondence with T. Vijayaraghavan, though he had never written a paper jointly with him. Also, through these letters, one may notice Pillai’s attempts to solve the famous open problems of that time like ‘Four Colour Problem’, ‘infinitude of integer polynomial values which are square-free’ and the distribution modulo 1 of the sequence $(3/2)^n$. Also, we could trace his contribution to ‘smooth numbers’ through one of the letters. Hence, we decided to include letters, which have mathematical content.

Although we have made every effort to correct the misprints in the published papers, we might have missed some. We have also included Mathematical Reviews for the papers that were reviewed critically.

First, we would like to thank Ramanujan Mathematical Society, Lecture notes series for coming up with the great idea of bringing out this collection with financial support from DST. Our special thanks to Prof. R. S. Kulkarni for showing great enthusiasm in this project. Also, we thank ‘the Society of Indian Academies’ in America for generous financial support to this project.

We would like to thank Mr. Chavadi A. Chockalingam Pillai, brother-in-law of Pillai, for providing valuable information regarding Pillai and making available the letters from various mathematicians and the unpublished works of Pillai. We also owe thanks to Pillai’s sons, Mr. S. Natarajan, Mr. S. Subramanian and Mr. S. Arunachalam for their valuable help.

We would like to thank Prof. J.-M. Deshouillers for writing the foreword. Also, we thank Prof. M. Waldschmidt and Prof. S. D. Adhikari for contributing articles on Pillai's work. We have included Prof. K. Chandrasekharan's article which was published in *Journal of Indian Mathematical Society* after Pillai's death in 1950, and the last two pages of an article on 'Ramanujan and I' by Prof. P. Erdős. We also would like to thank Prof. T. D. Wooley for writing comments on one of unpublished works of Pillai, namely, 'Waring's Problem – VII' and thank Prof. B. Sury for writing his comments on a paper of Pillai, namely, 'On the divisors of $a^n + 1$ '. Our special thanks to Prof. S. Kanemitsu for providing a beautiful sketch of Pillai.

We would like to acknowledge the help of V. O. C. College (Tutucorin) Correspondent Mr. A. P. C. V. Chockalingam Pillai, Librarian Ms. Esakki Ammal, Prof. P. Raina, Prof. G. Grekos, Prof. A. Geroldinger and Dr. S. Laishram in getting some published papers of Pillai. Our special thanks to Prof. C. Srinivasan, Madurai Kamaraj University, for introducing Chavadi A. Chockalingam Pillai to the second editor. Also, the second editor is thankful to Bhaskaracharya Pradishtana, Pune for their nice hospitality during his two visits regarding this project.

We greatly appreciate the enthusiasm and the help of many young colleagues namely, Dr. S. Archana Morye, Dr. B. Sahu, Mr. M. Chintamani, Mr. Karam Deo, Dr. Ritumoni Sharma, Dr. Anirban Mukhopadhyaya, Dr. N. Amora, Ms. R. Indra, Ms. Sunaina Seth, Ms. R. Ananthi, Mr. Bhanu Pratab Singh, Dr. Nishikanta Khandai and Dr. Tanusree Pal in the preparation of this work.

The second editor specially thanks Prof. M. Ram Murty, Prof. V. Kumar Murty and Dr. Purusottam Rath for their constant encouragement and many useful discussions related to this project and going through some of the unpublished papers of Pillai. Finally, we thank Prof. M. R. Modak for carefully going through all the unpublished works and letters in Volume II and for his many suggestions to improve the presentation.

Last but not the least, we would like to thank all our colleagues for their encouragement and help in various ways. Also, the references in the editor's note in this collection are not exhaustive. We might have missed a few references in-advertently. Hopefully, any omissions will be corrected in the next edition.

Chennai
Allahabad
22 December, 2009

R. Balasubramanian
R. Thangadurai

Foreword

When R. Balasubramanian mentioned his project of editing, with R. Thangadurai, the complete works of Subbayya Sivasankaranarayana Pillai, I was extremely touched and enthusiastic, and the more I thought about this project, the more enthusiastic I became.

I first heard the name of S. S. Pillai when, as a teenager, my interests turned to number theory and more specifically to additive questions and Waring's problem: this explains the emotional attachment I have to the project. As for the initial enthusiasm, it comes from the frustration I encountered a few years later when noticing the difficulty of finding some of the main papers of Pillai: we are taught to read classical, original papers, but finding them may be hard! Were it be for this reason alone, for making available the original and remarkable contribution of Pillai to different aspects of mathematics, mainly number theory, the editors of these volumes should be heartily thanked.

In fact, the enterprise of the editors goes far beyond the collection of the published papers of Pillai. It also includes recollections of his life and achievements by K. Chandrasekharan and P. Erdős, discussion of his contribution to Waring's problem, exponential diophantine equations and Ω -methods, by R. Balasubramanian, M. Waldschmidt and S. D. Adhikari, respectively. It also includes lectures given by Pillai as well as several texts which appear in print here for the first time. His correspondence, for instance, sheds light on his attempts to solve such major questions as the four colour problem, the square-free values in polynomial sequences, the distribution modulo 1 of the sequence $(3/2)^n$, Fourier series, . . .

This project also has another great merit. Our collective subconscious often reduces Indian mathematics to the invention of "0" and to "Ramanujan". No doubt that the latter deserves the unique place that he has in the Pantheon of the greatest mathematicians of all times. However, it is high time that the outstanding contribution of Indian mathematicians like S. S. Pillai emerge from his shadow: another reason for thanking R. Balasubramanian, R. Thangadurai and the Ramanujan Mathematical Society.

Bordeaux, France
12th October, 2009

Jean-Marc Deshouillers