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The Isoperimetric Problem in the Plane

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The purpose of this short article is to give an elementary proof of the following well-known theorem.

Main Theorem: *Of all the closed figures in the euclidean plane with a given perimeter, circle has the maximum area.*

Proof. We prove this in five stages.

Stage 1. Given the sides of length p and q of a triangle PQR and the angle R included by them variable, the area happens to be maximum whenever the angle R is a right angle.

It is known that, $\sin R$ is maximum when $R = 90$. So, the area of the triangle PQR which is equal to $\frac{1}{2}pq \sin R$ is maximum when $R = 90$.

Stage 2. Let us call the prospective figure F. Then if F has maximum area it has to be convex.

Theorem 1. *In simpler words it cannot have dents as shown in Figure 1, below.*

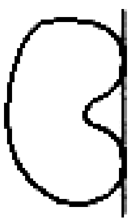

Figure 1.

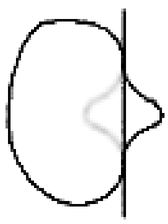

Figure 2.

Proof of Theorem 1. Assume contrary. In other words assume that F is of the form shown in Figure 1. We replace F by, say F_1 (Figure 2, the former part is shown faint). It is easy to see that F and F_1 have the same perimeter but F_1 has the larger area. Thus we have proved that F is convex.

Stage 3. Now let P, Q be points on the perimeter of F, (Figure 3), so that the parts PXQ and PYQ of the perimeter are equal in length. We describe this by saying that the points P and Q bisect the perimeter. We now assert

Theorem 2. *If the area happens to be maximum then segment PQ bisects the area F, (Figure 3).*

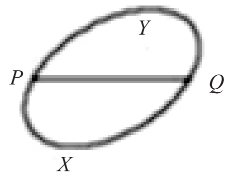

Figure 3.

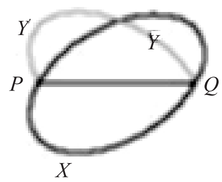

Figure 4.

Proof of Theorem 2. In other words, if F has maximum area and the arcs PXQ and PYQ have equal lengths then the areas of the Figures PXQ and PYQ are equal.

Again assume contrary and, for the sake of definiteness, that the area of PXQ is larger than that of PYQ. Then again replace the region PYQ by the mirror image $PY'Q$ of the region PXQ in PQ (Figure 4, the new region $PY'Q$ is shown faint). The claim that F has maximum area fails and the theorem 2 is established.

Stage 4. It is now enough to consider just the half part of F and show that it turns out to be a semicircle.

Theorem 3. *If the area PYQ is maximum then for any point R on the arc PYQ angle $PRQ = 90$.*

Proof of Theorem 3. Assume contrary and as the first case assume it to be acute (Figure 5). We keep the shape of the petal parts fixed and pull the points P, Q apart so as to make the angle $PRQ = 90$, (Figure 6). Note that the length of the arc PRQ does not change.

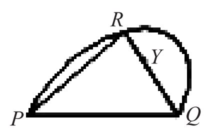

Figure 5.

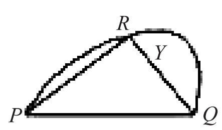

Figure 6.

Certainly the area of the triangle PQR thus obtained is larger than the one in Figure 6. In the second case where the angle PRQ happens to be obtuse one can pull the points P, Q closer so as to make the angle $PRQ = 90$.

Stage 5. Finally, this being the case for every point R on the arc PYQ the arc has to be a semicircle.

Thus the main theorem is established.

Poincaré Conjecture

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Abstract. Henri Poincaré asked, in the year 1904, whether every simply-connected closed three dimensional manifold is homeomorphic to the sphere $\mathbb{S}^3$. The assertion that it is so is known as Poincaré conjecture. It was finally solved a hundred years later by G. Perelman who was awarded the Fields Medal in the Madrid ICM-2006. In these notes I shall explain the conjecture and a generalization known as 'Thurston's geometrization conjecture', briefly mention Richard Hamilton's result and the core idea of Perelman's work.

1. Basic Notions

Let us begin by recalling the classification of (compact) surfaces. Recall that any compact connected surface (=two-dimensional manifold) S without boundary is determined upto homeomorphism by the its Euler characteristic and whether or not it is orientable. Indeed, if the surface S is orientable then its Euler characteristic $\chi(S)$ is an even number not exceeding 2. Writing $\chi(S) = 2 - 2g$, we see that the integer g is non-negative, called the *genus* of S. The surface S is homeomorphic to the 'sphere with g handles'; the case $g = 0$ corresponds to the sphere $\mathbb{S}^2 = \{v \in \mathbb{R}^3 \mid \|v\| = 1\}$. When $g = 1$, the surface S is just the torus $T := \mathbb{S}^1 \times \mathbb{S}^1$. One can describe the higher genus surfaces, that is, surfaces with genus $g \geq 2$, as a connected sum $T\#\cdots\#T$ of g copies of T. Thus we have a list $\mathbb{S}^2, T, T\#T, \ldots$, in which no two surfaces in the list are homeomorphic and any compact connected oriented surface is homeomorphic exactly one in this list. There is a similar list in the case of compact connected non-orientable surfaces: $\mathbb{P}, \mathbb{P}\#\mathbb{P}, \ldots, \mathbb{P}\#\cdots\#\mathbb{P}, \ldots$, where $\mathbb{P}$ is the projective plane obtained from $\mathbb{S}^2$ by identifying antipodal points $v, -v$ for each $v \in \mathbb{S}^2$. Thus we have a complete classification of compact surfaces.

One would like to have a result of this nature for higher dimensional manifolds. More precisely, topologists would like to have a collection $\{M_\alpha\}$ of (compact) n-dimensional manifolds upto homeomorphism such that (i) M_α is not homeomorphic M_β for $\alpha \neq \beta$, (ii) any n-dimensional manifold M is homeomorphic to exactly one of the M_α, and, (iii) given manifold M a way to decide (that is an algorithm that will decide) the M_α to which M is homeomorphic. It has been known as a consequence of some very remarkable results established in the 1950s concerning 'word problems' in the theory of group presentations that there cannot be any such classification theorem for manifolds of dimension four and higher. One might view Poincaré conjecture as a classification theorem for simply-connected 3-manifolds:

Poincaré Conjecture: If M is a compact simply connected 3-manifold, then M is homeomorphic to $\mathbb{S}^3$, the three-dimensional sphere.

It is known that any compact simply connected 3-manifold has the same *homotopy type* as that of $\mathbb{S}^3$. The question is whether such a manifold has the same topological type as $\mathbb{S}^3$. The following is a generalization of the above conjecture:

Generalized Poincaré Conjecture: If M is any compact n-manifold having the same homotopy type as the n-sphere $\mathbb{S}^n$, then M is homeomorphic to $\mathbb{S}^n$.

It should be remarked that there are 3-manifolds L_1 and L_2 such that L_1 and L_2 are homotopically equivalent but are not homeomorphic. One can take L_1, L_2 to be certain lens spaces.

Interestingly, the generalized Poincaré conjecture was established first for higher dimensions: For dimensions five and above it was established in 1959 by Stephen Smale who was awarded the Fields Medal in the year 1966 at the Moscow ICM. Affirmative solution in dimension 4 had to wait till 1982; it was proved by Michael Freedman who was awarded the Fields Medal in the year 1986 at ICM, Berkeley.

The original conjecture of Poincaré which was for dimension three was the hardest and had to wait for a hundred years to be settled completely!

If M and N are two manifolds which are homeomorphic, are they necessarily diffeomorphic? This is known to be true for surfaces. It was generally thought that this might be so until John Milnor proved in 1957 that the 7-sphere has exactly 28 distinct inequivalent differentiable structures. For this remarkable achievement he was awarded the Fields Medal in 1962. Milnor and Kervaire obtained examples of topological manifolds which do not admit any differentiable structures.

The situation in dimension three is, however, very special. It is known that any topological 3-manifold admits a unique differentiable structure. Thus two smooth 3-manifolds are homeomorphic if and only if they are diffeomorphic. It is still unknown whether $\mathbb{S}^4$ admits a unique differentiable structure. It should be mentioned here that $\mathbb{R}^n$, $n \neq 4$, is known to admit a unique differentiable structure. It was discovered by Simon Donaldson, who received the Fields Medal in 1986, that $\mathbb{R}^4$ admits 'exotic' differentiable structures.

We give here a brief exposition of some important developments arising out of efforts to classify three-dimensional manifolds. Some of the technical terms used in these notes are explained in the appendix.

2. Topology of 3-Manifolds

Recall that the Euler characteristic of a compact connected surface is a very powerful invariant. Just this number and the additional information concerning its orientability determines the topological type. It turns out that in the case of compact 3-manifolds—indeed any odd-dimensional manifolds—the Euler characteristic is always zero. Thus the Euler characteristic reveals nothing about the topology of the 3-manifold. However, the fundamental group turns out to be a very strong invariant for a compact connected 3-manifold M. For instance, when M is orientable it determines its *homology groups* completely. Indeed, $H_0(M) \cong H_3(M) \cong \mathbb{Z}$ for any (connected) oriented 3-manifold. Now $H_1(M)$ is isomorphic to the abelianised fundamental group $\pi_1(M)/[\pi_1(M), \pi_1(M)]$ by Hurewicz theorem. Now, by Poincaré duality, $H_2(M) \cong H^1(M) = Hom(H_1(M); \mathbb{Z})$. In particular, if $\pi_1(M)$ is trivial, then $H_1(M) = 0 = H_2(M)$ and so the manifold M has the same homology groups as $\mathbb{S}^3$. The converse however is not true. Poincaré himself constructed an orientable 3-manifold M which has the same homology groups as that of $\mathbb{S}^3$ but $\pi_1(M)$ is a certain finite subgroup J of $SU(2) \cong \mathbb{S}^3$ which is perfect. Indeed the manifold M can be described as the quotient $\mathbb{S}^3/J$. The group J has centre $Z(J)$ a group of order 2 and $J/Z(J) \subset SO(3)$ can be identified with the group of symmetries of the regular icosahedron. In fact, Poincaré asserted in 1901 that any closed 3-manifold having the same homology groups as the 3-sphere must be homeomorphic to the 3-sphere. He corrected himself by constructing the above example a few years later and asked the question whose affirmative answer is known as the Poincaré conjecture.

We shall now recall a few well-known statements which are known to imply or be equivalent to the Poincaré conjecture.

Suppose that M is a homotopy 3-sphere and S an imbedded 2-sphere in M cutting M along S we get two 3-manifolds-with-boundary, M_1, M_2 each of whose boundary is S. It can be shown that each M_i is contractible. If both M_1, M_2 are homeomorphic to the disk $D := \{x \in \mathbb{R}^3 \mid \|x\| \leq 1\}$, then it can be shown that M is *homeomorphic* to the 3-sphere.

A contractible 3-manifold-with-boundary which is *not* homeomorphic to the 3-disk D is called a fake 3-cell. Thus, Poincaré conjecture is equivalent to the following assertion: *There does not exist any fake* 3-*cell.*

J. H. C. Whitehead gave an example of an *open* (i.e., connected, non-compact) 3-manifold M which is contractible but not homeomorphic to the Euclidean space $\mathbb{R}^3$. (In dimension 2 every contractible open 2-manifold is homeomorphic to $\mathbb{R}^2$.) In the course of its construction, he made essential use of what is now known as the Whitehead link. It consists of two copies of imbedded circles in $\mathbb{R}^3$ which have linking number zero but yet they cannot be pulled apart—thus they are 'linked'. Whitehead constructed this example when he found a gap in his own 'proof' of the Poincaré conjecture.

We shall now recall a group theoretic reformulation of the Poincaré conjecture. View $\mathbb{S}^3$ as the one-point compactification $\mathbb{R}^3 \cup \{\infty\}$. Consider the 'standard' imbedding of the (orientable) genus g surface Σ, namely the 2-sphere with g-handles, in $\mathbb{R}^3 \subset \mathbb{S}^3$. The complement of Σ in $\mathbb{S}^3$ consists of two connected components whose closures M_1 and M_2 are compact 3-manifolds-with-boundary with boundary being Σ. In fact M_1 and M_2 are homeomorphic and the manifold $\mathbb{S}^3$ is obtained from M_1 and M_2 by identifying their common boundary Σ. M_1, M_2 are known as 'handle-bodies'. It is known that *any* 3-manifold admits handle-body decomposition $M = M_1 \cup_h M_2$—called Heegaard splitting—for suitable boundary identification h, which is a homeomorphism of the common boundary surface $h : \partial M_1 \longrightarrow \partial M_2$. The genus of the surface $\partial M_i =: \Sigma$ is called the genus of the Heegaard splitting. The homeomorphism type of the resulting 3-manifold depends not only on the genus g of the surface ∂M_1 but also on the homeomorphism h which prescribes the boundary identification $x \sim h(x), x \in \partial M_1$.

There is a notion of equivalence of Heegaard splittings of a given 3-manifold. Waldhausen has shown that any two Heegaard splittings of $\mathbb{S}^3$ over a surface of same genus are equivalent.

Suppose that $M_1 \cup_h M_2 = \mathbb{S}^3$ is a Heegaard splitting of $\mathbb{S}^3$ of genus g. By considering the induced map in fundamental groups of the inclusions $\Sigma \hookrightarrow M_i$ we obtain homomorphisms $\phi_i : \pi_1(\Sigma) \longrightarrow \pi_1(M_i)$, $i = 1, 2$ where Σ is the common boundary of $M_i \subset M$. These homomorphisms yield a homomorphism $\phi = (\phi_1, \phi_2)$, that is, $\phi : \pi_1(\Sigma) \longrightarrow F_g \times F_g, \phi(g) = (\phi_1(g), \phi_2(g))$ upon identifying $\pi_1(M_i)$ with the free group F_g of rank g. The homomorphism ϕ is called *the splitting homomorphism* associated to the Heegaard splitting.

Using Seifert-Van Kampen theorem, we see that M is simply connected if and only if ϕ is onto. In the case of the standard Heegaard splitting of $\mathbb{S}^3$, the homomorphisms $\phi_i, i = 1, 2$, have a simple and explicit description.

Jaco proved that starting with any surjective homomorphism $\phi = (\phi_1, \phi_2)$ where $\phi_i : \pi_1(\Sigma) \longrightarrow F_g$ where Σ_g is a closed orientable surface of genus g, one obtains a Heegaard splitting of a 3-manifold which is simply connected whose associated splitting homomorphism is ϕ.

We say that two homomorphisms $\phi, \psi : G \longrightarrow F_g \times F_g$ of groups are equivalent if there exist automorphisms $\alpha : G \longrightarrow G$ and $\beta_i : F_g \longrightarrow F_g$ such that $(\beta_1 \times \beta_2) \circ \psi = \phi \circ \alpha$; that is, the diagram below is commutative:

$$\begin{array}{ccc} G & \xrightarrow{\psi} & F_g \times F_g \\ \alpha \downarrow & & \downarrow \quad \beta_1 \times \beta_2 \\ G & \xrightarrow{\phi} & F_g \times F_g \end{array}$$

The following algebraic reformulation of Poincaré conjecture is well-known from the work of Waldhausen and the result of Jaco stated above. See [5] for further details.

Poincaré conjecture holds if and only if for any genus $g \geq 1$, any surjective homomorphism $\psi : \pi_1(\Sigma_g) \longrightarrow F_g \times F_g$ is equivalent to the splitting homomorphism ϕ associated to the standard Heegaard splitting of $\mathbb{S}^3$.

A 3-manifold M is called *prime* if $M = M_1 \# M_2$ implies that one of M_1, M_2 is the 3-sphere. M is said to be *irreducible* if every imbedded 2-sphere in M bounds a 3-cell. Thus M is prime if it is irreducible. If M is prime but not irreducible, then M is known to be a 2-sphere bundle over the circle. If $M = M_1 \# M_2$, then using Seifert-Van Kampen theorem one shows that the fundamental group of M is isomorphic to the free product of the fundamental groups of M_1 and M_2. In symbols, $\pi_1(M) \cong \pi_1(M_1) * \pi_1(M_2)$. In particular, M is prime if its fundamental group cannot be decomposed as a non-trivial free product.

A basic result in the study of 3-manifolds is the 'prime factorization':

Every closed connected orientable 3-manifold M can be expressed as a connected sum $M_1 \# \cdots \# M_k$ where each M_i is prime. This result was proved by H. Kneser in 1929. In general, the prime factorization is not unique: There exist closed non-orientable manifolds N such that $N \# N \cong N \# (\mathbb{S}^2 \times \mathbb{S}^1)$. However, Milnor showed that, for orientable 3-manifolds, prime factorization is unique (upto order of the summands and orientation preserving homeomorphisms). In view of prime factorization, if M is a homotopy 3-sphere (equivalently a simply connected closed 3-manifold) then all its prime factors are also homotopy 3-spheres. Many questions concerning 3-manifolds can be reduced to the case of prime manifolds.

Suppose that M is a closed connected 3-manifold and V is an imbedded closed surface of non-positive Euler characteristic. (It is not assumed that V is orientable.) We say that V is *incompressible* in M if the inclusion map $V \subset M$ induces a monomorphism of fundamental groups.

The basic idea of William Thurston in his formulation of *Geometrization Conjecture* is that any 3-manifold can be cut along spheres and incompressible tori or Klein bottles so that each of the resulting connected component has a nice geometric structure. We explain this more precisely in the next section.

3. Thurston's Geometrization Conjecture

Recall that in two dimensions, there are three Riemannian manifolds, $\mathbb{S}^2$, $\mathbb{R}^2$, and the Poincaré upper half-space $\mathcal{H}^2$, which are simply-connected, complete and have constant curvature 1, 0 and -1 respectively. Here $\mathcal{H}^2$ is the Poincaré upper half-space $\{z = x + \sqrt{-1}y \mid y > 0\} \subset \mathbb{C}$ with the Poincaré metric $ds^2 = (1/y^2)(dx^2 + dy^2)$. If S is any compact surface, it admits a Riemannian metric with constant curvature $1, 0$, or -1, depending on whether its Euler characteristic is positive, zero, or, negative. In each case the universal cover $\tilde{S}$ of S is $\mathbb{S}^2, \mathbb{R}^2$, or $\mathcal{H}^2$ respectively. The covering projection $\tilde{S} \longrightarrow S$ is a local isometry. Observe also that each of the spaces $\mathbb{S}^2, \mathbb{R}^2$ and $\mathcal{H}^2$ is homogeneous as a metric space: the group SO(3) of proper rotations of $\mathbb{R}^3$ acts transitively on $\mathbb{S}^2$, the group $\mathbb{R}^2$ acts transitively on itself by translations, and the group $\mathrm{PSL}(2, \mathbb{R}) := \mathrm{SL}(2, \mathbb{R})/\{\pm I\}$ acts transitively on $\mathcal{H}^2$ via Möbius transformations. ($\mathrm{SL}(2, \mathbb{R})$ denotes the group of 2×2 matrices over $\mathbb{R}$ with determinant 1.) These are the only *models* of geometries in 2-dimensions: the elliptic, the Euclidean or flat, and the hyperbolic, respectively. Observe that although $\mathbb{R}^2$ and $\mathcal{H}^2$ are *homeomorphic* (even diffeomorphic), they are very different as metric spaces.

In dimension 3, there are eight distinct model geometries. (It turns out that there are a few more geometries possible, but they do not give rise to compact quotients. See [15].) That is, there are eight simply-connected 3-dimensional Riemannian manifolds of whose group of isometries act transitively and which admit compact (or *finite volume*) quotients. They are $\mathbb{S}^3, \mathbb{R}^3, \mathcal{H}^3, \mathcal{H}^2 \times \mathbb{R}, \mathbb{S}^2 \times \mathbb{R}^1, Sol, Nil, \widetilde{\mathrm{SL}}(2, \mathbb{R})$. Of these, the first three in the list have constant sectional curvature $1, 0, -1$ respectively. Here *Sol* denotes the 2×2-upper triangular matrices over $\mathbb{R}$ with positive diagonal entries, Nil denotes the space of 3×3 unipotent upper triangular matrices, and, $\widetilde{\mathrm{SL}}(2, \mathbb{R})$ is the universal cover of $\mathrm{SL}(2, \mathbb{R})$. *Note that* $\mathbb{S}^3$ *is the only one in the list which is compact.*

A connected, complete Riemannian manifold 3-manifold M is called locally homogeneous if the universal cover $\tilde{M}$ of M is homogeneous. Thus $M = \tilde{M}/\Gamma$ where $\tilde{M}$ is one of the eight 3-manifolds listed above and $\Gamma = \pi_1(M)$ is a discrete subgroup of the group of isometries of $\tilde{M}$.

Thurston's Geometrization Conjecture: *Let M be a closed, oriented, prime* 3*-manifold. Then there exists an imbedding of a disjoint union of* incompressible 2*-tori and Klein bottles in M such that every component of the complement admits a Riemannian metric with respect to which it has finite volume and is locally homogeneous.*

The geometrization conjecture implies the Poincaré conjecture. To see this, let M be any prime 3-manifold with *finite* fundamental group. Then M has no incompressible tori or Klein bottles. By the geometrization conjecture, M has a Riemannian metric with respect to which it is locally homogeneous. Since $\pi_1(M)$ is finite, $\tilde{M}$ is compact, and hence, remarked earlier, $\tilde{M}$ has to be the 3-sphere.

The 3-manifolds which are quotients of $\mathbb{S}^3$ were classified by H. Hopf.

4. Ricci Flow

Let M be a connected smooth manifold of positive dimension. The set of all Riemannian metrics on M has the structure of a convex cone of an infinite dimensional vector space. Indeed, if g_0 and g_1 are any Riemannian metrics on M and if λ_0 and λ_1 are smooth positive functions on M then $g = \lambda_0 g_0 + \lambda_1 g_1$ is also a Riemannian metric.

Let $Ric(g)$ (or, simply, Ric) denote the Ricci curvature tensor determined by a Riemannian metric g. Recall that Ric is, like the Riemannian metric, a contravariant symmetric 2-tensor on the manifold M. The Ricci flow equation is an equation on the space of all Riemannian metrics on M. The *Ricci flow equation* on a Riemannian manifold (M, g), introduced by Richard Hamilton is:

$$\frac{\partial g(t)}{\partial t} = -2Ric(g(t)) \qquad (*)$$

with the initial condition $g(0) = g$.

A solution to the above equation is a curve $t \mapsto g(t)$ in the space of Riemannian metrics on M starting at $g = g(0)$ such that at any time t, the metric $g(t)$ flows or 'evolves' in the direction of $-2Ric(g(t))$. Suppose that g is an Einstein metric, i.e., the Ricci curvature tensor $Ric(g)$ is a multiple ag of g for some constant g. Then the solution to equation $(*)$ is a rescaling of g. If the multiple $a > 0$, the solution exists for a finite time T, after which the curvature becomes infinite and the manifold becomes extinct. In case $a < 0$ the solution exists for all times. It turns out that in dimension 3, if g is an Einstein metric on M, then the sectional curvature is constant and so M is locally homogeneous. Hamilton's idea is the following. Start with an arbitrary Riemannian metric $g = g(0)$ on M and allow the metric to vary so that the Ricci flow equation $(*)$ holds. Then, examine the limiting metric if it is (up to scaling) an Einstein metric. If it is so, then M is locally homogeneous. There are several problems to overcome. First, Hamilton showed that, if M is compact and $g = g(0)$ is arbitrary, then a solution to the Ricci flow equation always exists and is unique for $0 \leq t < \epsilon$ for some $\epsilon > 0$. If $T < \infty$ is the largest such that the solution exists for the time interval $[0, T)$, then there exists a point p on M at which the Ricci curvature tensor of the metric $g(t)$ becomes unbounded as $t \to T$. We say that 'a singularity develops at p'. For example, this happens in the case we start the Ricci flow with the standard metric on 3-sphere.

A Very Rough Plan of Perelman's Proof.

I can do no better than to indicate the major and very broad steps involved in Perelman's proof of Poincaré conjecture. The complete proofs can be found in [9].

- *Classification of types of singularities.* Singularities at finite time occurs in two types of regions. The first type of regions are those where the curvature becomes unbounded at all points in finite time (as in the case of the 3-sphere with its standard metric). The second type of regions are long thin tube (diffeomorphic to $\mathbb{S}^2 \times \mathbb{R}$). Perelman shows that at most *finitely many* singularities can develop at any finite time. Thus the singularities are all isolated at any given time.
- *How to continue the Ricci flow beyond the time when singularity develops?* Perelman performs surgeries in a controlled manner, so that, after the surgery (which in certain cases corresponds to connected sum decomposition), one starts all over again with the Ricci flow with the resulting new Riemannian manifold, which may not be connected. This way Perelman is able to proceed with the Ricci flow. This process proceeds either indefinitely (i.e., for all time $t < \infty$) or until *the manifold becomes extinct*!
- *Finiteness of extinction time.* In case one starts with a simply connected 3-manifold (or a manifold with finite fundamental group), after finitely many surgeries, each of which is along an embedded $\mathbb{S}^2$, one is left with possibly more than one connected component, each of which becomes *extinct* after a finite time. (Recall the case of the standard 3-sphere.)
- *Poincaré conjecture from finite extinction time.* If a simply connected 3-manifold becomes extinct in finite time in the evolution of Ricci flow with surgery, then Perelman shows that it has to be a 3-sphere. Since the surgery at each stage was along an imbedded $\mathbb{S}^2$, the the original manifold one started with must have been a connected sum of $\mathbb{S}^3$. Thus the original manifold itself must be diffeomorphic to $\mathbb{S}^3$. This establishes Poincaré conjecture.

Perelman's paper [13] also deals with the more general geometrization conjecture. Here the Perelman surgery corresponds to cutting up the manifold into locally homogeneous pieces along incompressible tori (and Klein bottles). The problem becomes more complicated because, unlike in the case of a simply-connected 3-manifold, the manifold does not become extinct in finite time during Ricci flow. Recently Morgan and Tian [10] have given complete and detailed proof of the geometrization conjecture. More precisely, the major part of work of Morgan and Tian involves establishing a a result, namely Theorem 7.4 of [13], stated but not proved in [13] which was crucial for the proof of the geometrization conjecture. Bessières et al. [1] have also given a proof having a more topological flavour, besides using Ricci flow techniques. In view of these recent developments, one may conclude that problem of the topological (and consequently diffeomorphism) classification of compact three-dimensional manifolds has now been solved completely!

5. Appendix

We recall here some basic notions concerning topology of manifolds.

An n-dimensional *topological* manifold M is a Hausdorff topological space which is *locally Euclidean*, that is, there exists an open covering $\{U\}$ such that each U is homeomorphic to an open subset of $\mathbb{R}^n$. We call a pair (U, ϕ) a *chart* if ϕ is such a homeomorphism. A collection of of charts $\{(U, \phi)\}$ is called an atlas. We shall further assume that M is metrizable (equivalently paracompact). Basic examples of manifolds are open subsets of the Euclidean space $\mathbb{R}^n$ and the unit sphere.

Let M be an n-dimensional manifold and let $\mathfrak{A} = \{(U, \phi)\}$ be an atlas. We say that $\mathfrak{A}$ is a *smooth structure* on the manifold M if, (i) for any two $(U, \phi), (V, \psi) \in \mathfrak{A}$, the map $\psi \circ \phi^{-1} \colon \phi(U \cap V) \longrightarrow \psi(U \cap V)$ is smooth, i.e., all mixed partial derivatives exist and are continuous, and (ii) $\mathfrak{A}$ is a maximal such collection. Starting with any atlas $\mathfrak{A}$ satisfying condition (i), one can always expand it to make it maximal in a unique way. For this reason, condition (ii) is not needed upon in order to define smooth structure on M.

A continuous map $f \colon M \longrightarrow N$ between two smooth manifolds $(M, \mathfrak{A}), (N, \mathfrak{B})$ is said to be smooth if, for any charts $(U, \phi) \in \mathfrak{A}, (V, \psi) \in \mathfrak{B}$ such that $f(U) \subset V$, the map $\psi \circ f \circ \phi^{-1}$ is smooth. Thanks to chain rule for differentiation, this definition does not depend on the choice of the charts $(U, \phi), (V, \psi)$.

If $f\colon M \longrightarrow N$ is smooth, one-to-one and onto, whose inverse is also smooth, then we say that f is a *diffeomorphism*. This is the basic notion of 'equivalence' in the study of smooth manifolds; thus we identify two manifolds if they are diffeomorphic to each other.

Suppose that $f\colon U \longrightarrow \mathbb{R}^n$, $x \mapsto (f_1(x), \ldots, f_n(x))$ is a smooth map where U is an open subset of $\mathbb{R}^{m+n}$. If q is a regular value (that is, the Jacobian matrix $J(f) = (\partial f_i/\partial x_j)$ of f has maximum rank at all points of $f^{-1}(q)$), then $M := f^{-1}(q)$ is in a natural way a smooth manifold of dimension m. Thus the unit sphere $\mathbb{S}^n = \{x \in \mathbb{R}^{n+1} \mid \|x\|^2 = 1\}$ is a smooth manifold of dimension n. Likewise one sees that the group $\mathrm{SL}(n, \mathbb{R})$ of $n \times n$ real matrices with determinant equal to 1 is a smooth manifold of dimension $n^2 - 1$.

A celebrated theorem of H. Whitney says that any smooth manifold M of dimension n can be imbedded as a submanifold of the Euclidean space $\mathbb{R}^{2n}$, that is M can be regarded as a subspace of $\mathbb{R}^{2n}$ where the inclusion map $M \hookrightarrow \mathbb{R}^{2n}$ is smooth.

The tangent space Let M be a smooth manifold of dimension n and let $p \in M$. A *tangent vector* to M at p is an operator which assigns to each smooth function $f\colon M \longrightarrow \mathbb{R}$ a real number $v(f)$ which satisfies the following properties:

(i) $v(f_1) = v(f_2)$ if f_1 and f_2 agree in some neighbourhood of p

(ii) $v(af + g) = av(f) + v(g)$, and,

(iii) *Leibnitz rule:* $v(fg) = g(p)v(f) + f(p)v(g)$ for any smooth functions f, g and $a \in \mathbb{R}$.

The set of all tangent vectors to M at p is a vector space denoted T_pM and is called the *tangent space* to M at p. The number $v(f)$ is the *directional derivative* of f along v.

The set $TM := \coprod_{p\in M} T_pM$ has the structure of a smooth manifold whose dimension is twice that of M such that the obvious projection map $\pi : TM \longrightarrow M$ is smooth. TM is called *the tangent bundle* of M. A smooth map $v : M \longrightarrow TM$ is called a vector field if $\pi \circ v = id_M$.

When M is an open subset of $\mathbb{R}^n$, any tangent vector at T_pM is the vector space $\left\{\sum_{1\le i\le n} a_i\partial_i|_p \mid a_i \in \mathbb{R}\right\}$ where $\partial_i|_p(f) = \frac{\partial f}{\partial x_i}(p)$. The tangent bundle TM is just $M \times \mathbb{R}^n$ where $\sum a_i\partial_i|_p$ is identified with $(p; a_1, \ldots, a_n) \in M \times \mathbb{R}^n$.

Suppose that M is imbedded as a smooth submanifold in $\mathbb{R}^d$. If $\sigma\colon (-\epsilon, \epsilon) \longrightarrow M$ is a smooth curve such that $\sigma(0) = p$, then the velocity vector $\sigma'(0) := \frac{d\sigma}{dt}|_{t=0} \in \mathbb{R}^d$ is the tangent vector to M at p defined as follows: if f is a smooth function on M, then $\sigma'(0)(f) = \frac{d(f\circ\sigma)}{dt}|_{t=0} \in \mathbb{R}$. Any tangent vector to M can be realised as a velocity vector of a suitable smooth curve and so that the space of all velocity vectors at p can be identified with the tangent space T_pM; in particular $T_pM \subset \mathbb{R}^d$.

If $M = f^{-1}(q)$ where q is the regular value of a smooth function $f\colon U \longrightarrow \mathbb{R}^m$, U open in $\mathbb{R}^{m+n}$, then the tangent space T_pM can be identified with the kernel of the linear map $\mathbb{R}^{m+n} \longrightarrow \mathbb{R}^m$ defined by the Jacobian at p, $J_p(f)$. Thus TM is identified with the subspace $\{(p, \sum_i a_i\partial_i|_p) \in \mathbb{R}^n \times \mathbb{R}^n = T\mathbb{R}^n \mid p \in M, J_p(f)(\sum_i a_i\partial_i|_p) = 0\} \subset M \times \mathbb{R}^n$.

For example, if $\mathbb{S}^n = f^{-1}(1)$ where $f\colon \mathbb{R}^{n+1} \longrightarrow \mathbb{R}$ is the smooth function $f(x) = x_0^2 + \cdots + x_n^2$. Observe that 1 is a regular value for f. Indeed the Jacobian of f is the column matrix $(2x_0, \ldots, 2x_n)^t$. The corresponding linear map $\mathbb{R}^{n+1} \longrightarrow \mathbb{R}$ is $u \mapsto 2u_0x_0 + 2u_1x_1 + \cdots + 2u_nx_n = 2u \cdot x$. The kernel of this linear map at $x \in \mathbb{S}^n$ is $\{u \in \mathbb{R}^{n+1} \mid u \cdot x = 0\}$ which is just the plane orthogonal to the vector $x \in \mathbb{R}^{n+1}$ (with respect to the standard innerproduct on $\mathbb{R}^{n+1}$.

Note that when M is an imbedded submanifold of a Euclidean space $\mathbb{R}^d$, the tangent space T_pM inherits in a natural way an innerproduct since T_pM is a vector subspace of the Euclidean space $T_p\mathbb{R}^d$ which is canonically identified with $\mathbb{R}^d$. As the point p varies on M the innerproduct varies 'smoothly'. This leads to the notion of a Riemannian metric on M.

A *Riemannian metric* on M is the choice of an innerproduct $\langle\cdot,\cdot\rangle_p$ on the tangent space T_pM for each $p \in M$ such that the innerproducts vary smoothly with respect to p. Smoothness with respect to p is the requirement that if $X, Y : M \longrightarrow TM$ are smooth vector fields on M, then $p \mapsto \langle X_p, Y_p\rangle_p$ be a smooth function on M.

An *affine connection* on a manifold M is an $\mathbb{R}$-linear operator ∇_X, associated to each smooth vector field X on M, on the vector space of all smooth vector fields of M satisfying the following axioms:

(1) $\nabla_X(aY + bZ) = a\nabla_XY + b\nabla_XZ$,

(2) $\nabla_X(fY) = X(f)Y + f\nabla_XY$, and,

(3) $\nabla_{fX+gY} = f\nabla_X + g\nabla_Y$,

for all smooth functions f, g on M, vector fields X, Y, Z on M and all $a, b \in \mathbb{R}$. In view of (3) above, it follows easily that $(\nabla_XY)_p = (\nabla_{X'}Y)_p \in T_pM$ if $X_p = X'_p$. Thus it is meaningful to define, for $v \in T_pM$, $\nabla_vY \in T_pM$ to be $(\nabla_XY)_p$ for any vector field X such that $X_p = v$.

If M is a Riemannian manifold and ∇ is an affine connection on M, we say that ∇ is *compatible with the Riemannian metric* if, for any three smooth vector fields X, Y, Z on M, one has

$$Z(\langle X, Y\rangle) = \langle \nabla_Z X, Y\rangle + \langle X, \nabla_Z Y\rangle.$$

A connection ∇ is said to be *symmetric* if $\nabla_X Y - \nabla_Y X = \nabla_{[X,Y]}$. On a given Riemannian manifold, there exists a unique symmetric connection ∇ which is compatible with the metric. It is called the *Levi-Civita* connection.

Let ∇ be the Levi-Civita connection on M. Define the *curvature tensor* R of ∇ as follows: $R(X, Y) = \nabla_X \nabla_Y - \nabla_Y \nabla_X - \nabla_{[X,Y]}$.

Let X and Y be smooth vector fields on M and for each $p \in M$ we define an endomorphism $T_pM \longrightarrow T_pM$ where $v \mapsto R(X_p, v)(Y)$. Taking trace of this endomorphism gives a smooth function $p \mapsto Trace(R(X_p, v)(Y))$. $Ric(X, Y)$ is symmetric and bilinear in X, Y and so $(X, Y) \mapsto Ric(X, Y)$ is a (0, 2)-tensor and is called the Ricci tensor.

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Non-Locally Finite Packings in the Plane

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1. Introduction

Consider the following problem: *is it possible to construct non–empty, open, congruent sets $E_1, E_2, \ldots$ in the plane which are mutually disjoint, but which accumulate somewhere in the plane*? Of course, we have to specify which plane we are talking about, and here we shall only consider the Euclidean plane and the hyperbolic plane. In fact, it is very easy to construct such sets in each of these planes. However, if we add the requirement that the E_j be convex, then it is impossible to construct such sets in the Euclidean plane, but it is possible to do so in the hyperbolic plane. In this article we shall attempt to explain why this is so.

We shall assume that the reader is familar with the basic facts concerning the upper half–plane model $\mathbb{H} = \{x + iy : y > 0\}$ of the hyperbolic plane. The metric in $\mathbb{H}$ is given by $ds = |dz|/y$, and the conformal isometries of $\mathbb{H}$ are the Möbius transformations $z \mapsto (az + b)/(cz + d)$, where a, b, c and d are real, and $ad - bc > 0$. The geodesics in $\mathbb{H}$ are the vertical Euclidean half–lines and the Euclidean semicircles in $\mathbb{H}$ whose (Euclidean) centres lie on $\mathbb{R}$. A set E in $\mathbb{H}$ is (hyperbolically) convex if and only if each pair of points in E can be joined by a geodesic segment which also lies in E.

The hyperbolic plane is usually introduced to show the existence of a geometry in which the Parallel Axiom fails. However, it has many other features which differ markedly from the the Euclidean plane, and which arise because the hyperbolic metric has negative curvature. Informally, this means that two geodesics separate more rapidly in the hyperbolic plane than they do in the Euclidean plane, and this implies that, in some sense, there is much more space near infinity in the hyperbolic plane than there is in the Euclidean plane. As evidence of this we can look at the rate of change of the area $A(r)$ of a disc of radius r with respect to r in each of the geometries. In the Euclidean case, $dA(r)/dr = 2\pi r$, whereas in the hyperbolic case, $A(r) = 4\pi \sinh^2(\frac{1}{2}r)$ and $dA(r)/dr = 2\pi \sinh r$. In some sense, we have the different answers in the two geometries because $2\pi \sinh r$ grows much faster than $2\pi r$.

2. Two Constructions

We begin by showing that such a construction is possible in the Euclidean plane if we do not insist that the sets E_j be convex. Let C_0 be the part of the curve $y = 1/x$ in the first quadrant of the Euclidean plane. Now, for $n = 1, 2, \ldots$, let C_n be the translation of C_0 by the negative integer $-n$, and let E_n be the region in the upper half–plane plane which is bounded by C_n and C_{n+1}. Clearly, the sets E_n are mutually disjoint, open, non–empty, and congruent to each other. Moreover, the sets E_n accumulate (as $n \to \infty$) along the entire real axis.

Our second construction is in the hyperbolic plane $\mathbb{H}$. For $n = 0, 1, 2, \ldots$, let

$$E_n = \{x + iy \in \mathbb{H} : 2^{-(n+1)} < x < 2^{-n}\}.$$

This is a (hyperbolically) convex subset of $\mathbb{H}$, and the sets E_n are congruent to each other because, for each n, E_n is mapped onto E_0 by the hyperbolic isometry $z \mapsto 2^n z$. It is clear that these sets accumulate along the entire positive imaginary axis in $\mathbb{H}$.

Actually, much more is true. It is a remarkable fact that the hyperbolic plane can be *tesselated* by congruent, convex pentagons of area 2π in such a way that infinitely many of these pentagons accumulate at every point of the imaginary axis (see A. F. Beardon, *The Geometry of Discrete Groups*, Springer–Verlag, 1983, pp. 210–213).

3. The Euclidean Case

Here we work in the complex plane $\mathbb{C}$ and show that the construction is not possible if we insist the the sets E_j be convex.

Theorem 3.1. *Let $E_1, E_2, \ldots$ be mutually disjoint, congruent sets in $\mathbb{C}$, where each E_n is non–empty, open and convex. Then any bounded subset of $\mathbb{C}$ meets only finitely many of the E_n.*

It is sufficient to prove the following lemma in which $\overline{\mathbb{D}}$ is the closed unit disc.

Lemma 3.2. *Let E be an open, non-empty, convex set that contains $\overline{\mathbb{D}}$. Then there is a positive α such that, for all w in E, $E \cap \{z : |z - w| < 1\}$ has area at least α.*

Theorem 3.1 follows immediately from Lemma 3.2. First, the validity of Theorem 3.1 is unaltered if we rescale and translate the complex plane, so we may assume that E_1 in Theorem 3.1 contains $\overline{\mathbb{D}}$. We now apply Lemma 3.2 with $E = E_1$, and we obtain the positive number α with the given property. Since the E_n are congruent to each other, the same property holds for all E_n, *with the same* α. It is now clear that Theorem 3.1 follows from this, since if D is any disc of radius r, and if N of the sets E_n meet D, then, as the E_j are mutually disjoint, $N\alpha \leq \pi(r+1)^2$. Thus D cannot meet infinitely many of the E_n. It remains to prove Lemma 3.2, and to do this we need Lemma 3.3 (which applies to unbounded sets).

Lemma 3.3. *Let E be an open, non-empty, convex set that contains $\overline{\mathbb{D}}$. Suppose that E contains points z_n with $z_n = r_n e^{i\theta_n}$, where $r_n \to +\infty$ and $e^{i\theta_n} \to 1$ as $n \to \infty$. Then E contains the half–strip $S = \{t + is : t > 0,\ -1 < s < 1\}$.*

Proof. Let T_n be the closed triangular region with vertices i, $-i$ and z_n. By convexity, $T_n \subset E$. Note that

$$T_n = \{\lambda i + \mu(-i) + \nu z_n : \lambda, \mu, \nu \geq 0,\ \lambda + \mu + \nu = 1\},$$

as $\lambda i + \mu(-i) + \nu z_n$ is the centre of gravity of point masses λ at i, μ at $-i$, and ν at z_n.

We write $z_n = x_n + iy_n$. Since $e^{i\theta_n} \to 1$, we may assume that $x_n > 0$ for all n. Also, $x_n \to +\infty$, and $y_n/x_n \to 0$. Now take any $u + iv$ in S; that is, with $u > 0$ and $-1 < v < 1$, and let

$$\lambda = \frac{1}{2}\left(1 + v - \frac{u(1+y_n)}{x_n}\right),$$

$$\mu = \frac{1}{2}\left(1 - v - \frac{u(y_n - 1)}{x_n}\right), \quad \nu = \frac{u}{x_n}.$$

Of course, these values of λ, μ and ν have been obtained from a preliminary calculation which is omitted here. Now

(i) for sufficiently large n, λ, μ and ν are positive;
(ii) $\lambda + \mu + \nu = 1$, and
(iii) $\lambda i + \mu(-i) + \nu z_n = u + iv$.

This shows that $u + iv \in T_n$ for some n, and hence that $S \subset E$ as required.

Proof. [The proof of Lemma 3.2] For each w, let $\mathbb{D}(w) = \{z : |z - w| < 1\}$, and let $A(w)$ be the area of $E \cap \mathbb{D}(w)$. We want to show that $\inf_{w\in E} A(w) > 0$, and we argue by contradiction. Suppose that the conclusion is false; then there are points w_n in E such that $A(w_n) \to 0$. By passing to a subsequence we may assume that either (i) $w_n \to \zeta$ for some ζ in $\mathbb{C}$, or (ii) $w_n \to \infty$.

Case (i). $w_n \to \zeta$

Let Σ_n be the convex hull of $\mathbb{D} \cup \{w_n\}$, and Σ be the convex hull of $\mathbb{D} \cup \{\zeta\}$. Then

$$A(w_n) = \text{area}(E \cap \mathbb{D}(w_n)) \geq \text{area}(\Sigma_n \cap \mathbb{D}(w_n)) \to \text{area}(\Sigma \cap \mathbb{D}(\zeta)) > 0,$$

and this contradicts the assumption that $A(w_n) \to 0$. The justification for the limit here is as follows. For any set Q, let F_Q be the characteristic function of Q. Then $F_{\Sigma_n} \to F_\Sigma$ almost everywhere on $\mathbb{C}$. Lebesgue's Bounded Convergence Theorem now implies that

$$\text{area}(\Sigma_n \cap \mathbb{D}(w_n)) = \int_{\mathbb{C}} F_E F_{\Sigma_n} \to \int_{\mathbb{C}} F_E F_\Sigma = \text{area}(\Sigma \cap \mathbb{D}(\zeta)).$$

Case (ii). $w_n \to \infty$

We write $w_n = r_n e^{i\theta_n}$, where $r_n \to +\infty$, and, by passing to a subsequence, we may assume that $e^{i\theta_n}$ converges. Clearly (by rotating the plane) we may assume that $e^{i\theta_n} \to 1$ as $n \to \infty$; then, by Lemma 3.3, E contains the half–strip $S = \{x + iy : x > 0,\ |y| < 1\}$. Now $\Re[w_n] > 0$ (for sufficiently large n), and since E is convex it contains the convex hull of $S \cup \{w_n\}$. Thus, for example, if $w_n = u_n + iv_n$, $u_n > 0$ and $v_n > 0$, then E contains the region $W_n = \{x + iy : x > u_n,\ -1 < y < v_n\}$. A similar statement holds when $v_n \leq 0$, and we conclude that, for sufficiently large n, $\mathbb{D}(w_n) \cap E$ covers at least one quarter of the disc $\mathbb{D}(w_n)$. Since this implies that area$(\mathbb{D}(w_n) \cap E) \geq \pi/4$, this is a contradiction.

4. The Conclusion

We recall the remarks made in Section 1.. Suppose that (in either geometry) the mutually disjoint congruent sets $E_1, E_2, \ldots$ meet a given compact disc Q. Then $\sum_n \text{area}(Q \cap E_n)$ converges so that area$(Q \cap E_n) \to 0$. Intuitively, this means that as n increases, the sets $Q \cap E_n$ must look increasingly like thinner and thinner parallel strips. This forces the 'wider' part of E_n to move further and further away from Q, and as these sets are disjoint we must have sufficient room near ∞ to allow this to happen. This is why it is possible to construct such sets in the hyperbolic plane but not in the Euclidean plane.

Geometry, Topology and Dynamics in Negative Curvature

August 2–7, 2010

Venue: Raman Research Institute, Bangalore, India.

Description: This is a satellite conference for the 2010 ICM. The conference will focus on mathematics centered around negatively curved (or more generally, non-positively curved) spaces.

Contact: C. S. Aravinda at aravinda@math.tifrbng.res.in

For Further Information Visit:

http://www.icts.res.in/program/gtdnc

Third International Conference on Boundary Value Problems, Integral Equations and Related Problems

August 20–25, 2010

Venue: Beijing and Baoding, Hebei, China.

Topics: The conference will be about the following six subjects:

1. Various boundary value problems for partial differential equations and functional equations;
2. The theory and methods of integral equations and integral operators including singular integral equations;
3. Applications of boundary value problems and integral equations to mechanics and physics;
4. Numerical methods of integral equations and boundary value problems;
5. Theory and methods for inverse problems of mathematical physics;
6. Clifford analysis and some related problems with above subjects.

For Detailed Information, Visit/Contact:

http://www.math.pku.edu.cn/3inter.conf-bvp.ie.rps

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Differential Geometry and Its Applications

August 27–31, 2010

Venue: Masaryk University, Faculty of Science, Brno, Czech Republic, Europe.

Description: The DGA conferences take place regularly at one of the Czech universities every three years; the tenth conference in the series took place in Olomouc in 2007.

For Further Information Visit:

http://dga.math.muni.cz/dga2010

Workshop on Operator Algebras and Conformal Field Theory

August 16–21, 2010

Venue: University of Oregon, Eugene, Oregon.

Description: This will be an educational workshop aimed at exploring the foundations of conformal field theory from the perspective of operator algebras. It will focus on Wasserman's construction of CFTs associated to the loop group SU(n), which he uses to study representations of central extensions of the loop group.

For Further Information Visit:

http://www.uoregon.edu/∼njp/cft.html

40th Probability Summer School

July 4–17, 2010

Venue: Saint-Flour, France.

Description: Founded in 1971, this school is intended for Ph.D. students, teachers and researchers who are interested in probability theory, statistics, and in applications of these techniques. The three 12-hour courses of this year will be given by Franco Flandoli, Giambattista Giacomin, and Takashi Kumagai.

For Further Information Visit:

http://math.univ-bpclermont.fr/stflour/

11th International Conference on *p*-Adic Functional Analysis

July 5–9, 2010

Venue: Universite Blaise Pascal, Les Cezeaux, Aubiere, France.

Description: The conference follows 10 previous ones held each even years, the last one at Michigan State University (July 2008). It will gather specialists of various domains in *p*-adic analysis: spaces of functions, umbral calculus, Banach and Hilbert spaces, *p*-adic measures, Levi-Civita fields, *p*-adic Weyl algebras, Banach algebras, multiplicative spectrum, differentiable maps and differential manifolds on *p*-adic fields, analytic functions on a complete algebraically closed field, *p*-adic dynamical systems, *p*-adic Nevanlinna theory.

For Further Information Visit:

`alain.escassut@math.univ-bpclermont.fr`

International Conference: The Sixth Dynamical Systems and Applications-2010

July 10–14, 2010

Venue: Sea Life Hotel, Antalya, Turkey.

Aims and Topics of the Conference: The main aim of this conference is to provide impetus, motivation, and to bring together researchers and scientists working in the fields of Dynamical Systems and Applications by providing a forum for the academic exchange of ideas and recent research works. The proposed technical program of the conference will include contributed talks and keynote lectures. The areas of interest include but are not limited to: Ordinary and Partial Differential Equations, Difference Equations and Applications, Analysis and Applications, Applied Mathematics and Dynamical Systems.

For Further Information Visit:

`http://faculty.uaeu.ac.ae/hakca/Antalya-Dynamic-Systems-2010/Antalya.htm`

25th Summer Conference on Topology and Its Applications

July 25–30, 2010

Venue: Jan Kochanowski University in Kielce, Poland.

Description: The 25th Summer Topology Conference is an annual event bringing together an international audience of researchers in general topology and related fields. There will be special sessions in the following subjects: general/set-theoretic topology, geometric topology, continuum theory, dynamical systems, topological algebra, topology in computer science, topology in functional analysis.

For Further Information Visit:

`http://www.ujk.edu.pl/~topoconf/`

The XXI School of Algebra

July 25–31, 2010

Venue: Brasília, Brazil.

Description: The XXI School of Algebra, the 21st edition of the biannual Brazilian meeting of Algebraists, will take place at the University of Brasília, from July 25 to 31, 2010.

For Further Information Visit:

`http://www.algebra.unb.br`

The International Congress of Mathematicians ICM2010

ICM2010 hosts the following satellite conferences. For the complete details and additional information, we refer to

`http://www.icm2010.org.in/scientific-program/satellite-conferences`

CIMPA School of Number Theory in Cryptography and Its Applications

July 12–31, 2010

Venue: Kathmandu University, Nepal

Contact Person: F. Pappalardi at `pappa@mat.uniroma3.it`

K. Jha at `jhakn@ku.edu.np, jhaknh@yahoo.co.in`

Geometry, Topology and Dynamics of Character Varieties

July 19–23 (Workshop) and
August 10–14 (Conference) 2010

Venue: National University of Singapore

Modular Forms

August 1–17, 2010

Venue: Mahabalipuram, Tamil Nadu

Contact: M. Manickam at murugumanick@yahoo.com

Functional Analysis and Operator Theory

August 8–11, 2010

Venue: I.S.I., Bangalore

Contact: T. S. S. R. K. Rao and G. Misra at ramanuj@isibang.ac.in

Geometric Group Theory

August 9–14, 2010

Venue: Goa University

Contact: B. Sury and Pallavi Dani at pdani@math.lsu.edu

Operator Algebras

August 9–13, 2010

Venue: IMSc., Chennai

Contact: V. S. Sunder at sunder@imsc.res.in

Galois Representations in Arithmetic and Geometry

August 10–13, 2010

Venue: International Centre, Goa

Contact: John Coates, C. S. Dalawat at dalawat@gmail.com, Anupam Saikia at a.saikia@iitg.ernet.in and R. Sujatha

Algebraic and Combinatorial Approaches to Representation Theory

August 12–16, 2010

Venue: IISc, Bangalore

Contact: S. Viswanath at svis@math.iisc.ernet.in

Algebraic Geometry

August 13–16, 2010

Venue: University of Hyderabad

Contact: Jaya N. Iyer and V. Lakshmibai at icmag2010@yahoo.com

Geometric Topology and Riemannan Geometry

August 12–15, 2010

Venue: IISc., Bangalore

Contact: S. Gadgil at gtrg2010@gmail.com, siddhartha.gadgil@gmail.com, H. Seshadri and B. Datta

Integrable Systems and Geometry

August 12–17, 2010

Venue: Pondicherry University

Contact: K. M. Tamizhmani at tamizh@yahoo.com

Recent Trends in Graph Theory and Combinatorics

August 12–15, 2010

Venue: CUSAT, Cochin

Contact: A. Vijayakumar at icrtgc2010@cusat.ac.in, and at icrtgc2010@gmail.com

Mathematical Logic and Set Theory

August 13–15, 2010

Venue: IMSc., Chennai

Contact: R. Ramanujam at jam@imsc.res.in

Automorphic Forms and Number Theory

August 13–17, 2010

Venue: International Centre, Goa

Contact: E. Ghate and D. Prasad at icmgoa10@math.tifr.res.in

PDE and Related Topics

August 13–17, 2010

Venue: TIFR-CAM, Bangalore

Contact: M. Vanninathan at icmpde@math.tifrbng.res.in

Probability and Stochastic Processes

August 13–17, 2010

Venue: ISI, Bangalore

Contact: Abhay Bhatt at abhay@isid.ac.in, and Siva Athreya at athreya@isibang.ac.in

Quantum Probability and Related Topics

August 14–17, 2010

Venue: JNCASR, Bangalore

Contact: K. B. Sinha, Tirthankar Bhattacharyya and B. V. Rajarama Bhat at qpconference2010@gmail.com

Quantum Systems

August 14–18, 2010

Venue: IMSc., Chennai

Contact: A. Mohari at anilesh@imsc.res.in, and M. Krishna at krishna@imsc.res.in

Application of Control Theory and Optimization in Biochemical Pathways

August 15–17, 2010

Venue: CCMB, Hyderabad

Contact: M. Vidyasagar, and Somdatta Sinha at ICM2010satmeet@gmail.com

Mathematics in Science and Technology

August 15–17, 2010

Venue: India Habitat Centre, New Delhi

Algebraic and Probabilistic Aspects of Combinatorics and Computing

August 29 – September 3, 2010

Venue: Bangalore

Contact: C. R. Subramanian at crs@imsc.res.in

Analytic and Combinatorial Number Theory

August 29 – September 3, 2010

Venue: IMSc., Chennai

Contact: A. Mukhopadhyay, and K. Srinivas at icmzeta@imsc.res.in, adhyay@gmail.com

Buildings, Finite Geometries and Groups

August 29–31, 2010

Venue: ISI, Bangalore

Contact: N. S. Narasimha Sastry at bfgg@isibang.ac.in

Various Aspects of Dynamical Systems

August 29 – September 1, 2010

Venue: MS University, Vadodara, Gujarat

Contact: V. Kannan, and Tarun Das
E-mail: icm.ds2010@gmail.com, vksm@uohyd.ernet.in, and tarukd@gmail.com

Mathematics in Ancient Times

August 29 — September 1, 2010

Venue: Kerala School of Mathematics, Calicut

Contact: ttc icmatcal@gmail.com

Harmonic Analysis

August 29 – September 2, 2010

Venue: NISER, Bhubaneswar

International Workshop on Recent Advances in Computational Fluid Dynamics

August 30 – September 2, 2010

Venue: IIT Guwahati

Contact: racfd@iitg.ernet.in

Rings and Near Rings

August 30 – September 11, 2010

Venue: NEHU, Shillong, Meghalaya

Contact: M. Rege at mb29rege@yahoo.co.in

Probability and Statistics

September 1–3, 2010

Venue: Sambalpur University

Contact: S. K. Acharya

National Workshop Cum Training Programme on Recent Trends in Fluid Mechanics (NWCTP-RTFM)

July 06–12, 2010

Venue: Banaras Hindu University, Varanasi

The DST-Centre for Interdisciplinary Mathematical Sciences (CIMS) and the Department of Mathematics, Faculty of Science, Banaras Hindu University, Varanasi are organizing a one week's National Workshop cum Training Programme on Recent Trends in Fluid Mechanics between July 06–12, 2010 to train the research scholars and young teachers/scientists from various Universities/Colleges/Institutations in Fluid Mechanics and the related areas, and make them familiar with the recent development in this field.

Contents:

1) Magnetohydrodynamics
2) Non-Newtonian fluids
3) Flow and Dispersion through and past a porous medium
4) Convection: Heat and Mass Transfer
5) Smart materials of Nano structure
6) Linear and Non-linear stability of flows.

Contact Address:

B. S. Bhadauria (Convener)
NWCTP-RTFM, Department of Mathematics
Faculty of Science, BHU,
Varanasi
Pin 221005, Mobile No.: 09453641182

International Workshop on Graph Coloring and Its Applications – IWGCA 2010

August 07–09, 2010

Venue: Tiruchirappalli, India

International Workshop on Graph Coloring and its Applications – IWGCA 2010, is scheduled to be held on August 07–09, 2010 at Tiruchirappalli, India.

"Graph coloring" is one of the oldest and best-known areas of graph theory. As people became accustomed to applying the tools of graph theory to the solutions of the real world technological and organizational problems, new chromatic (that is, coloring) models emerged as a natural way of tackling many practical situations. "Internet statistics shows that graph coloring is one of central issues in the several hundred classical combinatorial problems". The reason of course lies in the simplicity of the formulation of problems on the one hand and numerous potential applications on the other, whenever such problems turn out to be tractable.

Contact Address:

Prof. R. Balakrishnan
Adjunct Professor
Department of Mathematics,
Bharathidasan University,
Tiruchirappalli 620 024, Tamil Nadu, India
Phone: 0431-2407065
E-mail: iwgc2010@yahoo.com

ICM 2010 – Satellite Conference International Workshop on Harmonic and Quasiconformal Mappings (HQM2010)

August 09–17, 2010

First Announcement

Organized by: Department of Mathematics, Indian Institute of Technology Madras, Chennai 600 036, India.

- **About the Workshop:** The workshop features short courses by about five experts in the field of p-harmonic mappings, with the aim of introducing the young Indian mathematicians to this field and to foster international research collaboration. Professors Leonid Kovalev, Jani Onninen, and Juha Kinnunen are among the experts who are expected to give some lectures each during the workshop. The subsequent lectures will focus on the related area of planar harmonic morphisms and quasiconformal mappings with the aim of introducing the young researchers to this field and to foster international research collaboration.

  This program is divided into two parts, the workshop (August 09–17, 2010) and the conference (August 16–17, 2010).

- **Objectives:** The workshop also aims to promote research collaborations between the various participants, and to introduce the study of (non-linear) p-harmonic mappings to the participants. The experts in this field will each give 4–5 lectures in the mornings, followed by question and discussion sessions in the afternoons of the workshops. The workshop will mainly focus more on the well-developed theory of (linear) harmonic mappings and planar quasiconformal mappings, thus enabling the participants to compare the non-linear setting with the linear setting and helping to identify possible features present in the non-linear p-harmonic mappings.

  The proceedings of the workshop will be published as a special issue by a reputed journal publisher, and will contain papers submitted by the participants of the workshop as well as a synopsis of the discussions conducted during the workshop.

- **List of Topics:** The principal focus of the workshop is the study of p-harmonic mappings. Falling under the broad category of Geometric Analysis, the linear analog ($p = 2$) is now currently being studied and its connection to quasiconformal and quasiregular mappings in planar domains is well-understood. The non-linear version, called p-harmonic mappings, is not so well-understood. The following is a preliminary list of topics to be covered in the workshop and conference:

  1. p-harmonic functions and mappings in plane and space
  2. Quasiconformal mappings and quasiregular mappings
  3. Hyperbolic type geometry and geometric function theory
  4. Analysis and function theory in metric measure spaces

- **About the Conference:** Apart from the above mentioned workshop there will be a conference having invited talks, parallel sessions to encourage the researchers, oral/poster paper presentations and problem discussion sessions during August 16–17, 2010. The topics include the theory of univalent functions and modern topics in geometric function theory.

- **Call for Papers:** There will be limited paper presentation sessions by participants and the accepted original papers will be considered for possible publication by a reputed journal publisher. Acceptance for presentation of a paper in the workshop does not guarantee the acceptance of the paper for publication. All full papers submitted will be scrutinized by a compatible referee and his/her opinion will be final for publication.

Updated information about the publication of the proceedings will be informed in due course.

- **List of Main Speakers Include:**

Tomasz Adamowicz (University of Cincinnati, USA)
Daoud Bshouty (Technion – Israel Institute of Technology, Israel)
Michael Dorff (Brigham Young University, USA)
Juha Kinnunen (Helsinki University of Technology, Finland)
Pekka Koskela (University of Jyvaeskylae, Finland)
Leonid Kovalev (Syracuse University, USA)
Abdallah Lyzzaik (American University of Beirut, Lebanon)
Miodrag Mateljevic (University of Belgrade, Serbia)
Jani Onninen (Syracuse University, USA)
Nageswari Shanmugalingam (University of Cincinnati, USA)
Toshiyuki Sugawa (Tohoku University, Japan)
Matti Vuorinen (University of Turku, Finland)
Brock Williams (Texas Tech University, USA)

Some Other Speakers: Some other speakers will give some pre-workshop lectures, etc... These will be announced in due course in detail.

Venue: IC & SR Building/Department of Mathematics Indian Institute of Technology Madras Chennai 600 036, India.

For Further Details/Registration Form, etc. can be Obtained From:

http://mat.iitm.ac.in/HQM2010;
http://sites.google.com/site/hqm2010iitmadras

Contact Details:

S. Ponnusamy
Convener (HQM2010)
Department of Mathematics
Indian Institute of Technology Madras
Chennai 600 036, India
Phone: +91-44-2257 4615 (Office)
+91-44-2257 6615 (Home)
Fax: +91-44-2257 4602 (Dept. Office)
Email: samy@iitm.ac.in; hqm2010@iitm.ac.in (Official)

Advanced Training in Mathematics Schools

Supported by
National Board for Higher Mathematics

Advanced Instructional School Schemes & Cohomology

28 June – 16 July, 2010

Venue: KSOM, Kozhikode

Conveners: A. J. Parameswaran & V. Balaji

A Brief Description of ATM Schools: Advanced Training in Mathematics (ATM) Schools are a joint effort of a large number of mathematicians in the country for training mathematics research scholars and teachers with generous support from the National Board for Higher Mathematics. The programmes are conducted in reputed mathematics departments in Summer and Winter each year. In these Schools, the emphasis is on problems solving and on understanding interrelations of basic subjects in mathematics. At the initial stage, ATM Schools consist of two Annual Foundation Schools (AFS I & II) in algebra, analysis, and topology. At a later stage, Advanced Instructional Schools (AIS) and workshops (ATMW) in all major areas are organised. Several advanced instructional schools (ATML) are organized each year exclusively for young lecturers in colleges and universities

Advanced Instructional School on Schemes & Cohomology: The AIS will be directed towards making young researchers in India familiar with the developments in Algebraic Geometry, with special emphasis on Grothendieck's programme. The three week programme is primarily aimed at early researchers in this field to become familiar and users of the tools and techniques in this subject. To acquire a good knowledge of modern algebraic geometry it is essential to see the "local" aspects coming from Commutative algebra in its interplay in geometry as well as the immense machinery of cohomology. These two themes will be the main ones for the workshop. To give a flavour of the manner in which these tools have lead to fundamental theorems in algebraic geometry, the final week will stress on some research themes. A team of active researchers in the field has been invited for this purpose. The basic reference for this course will be the book by

R. Hartshorne (Algebraic Geometry, Springer Graduate Texts in Mathematics) supplemented by other books by D. Mumford (The Red Book of Varieties and Schemes, Springer), Kenji Ueno (Algebraic Geometry, Parts 1, 2 and 3, American Mathematical Society Translations) and a few others.

List of Topics: Sheaves, schemes, elementary properties of schemes, morphisms, invertible sheaves and bundles, differentials, valuative criterion, Bertini's theorems, Lefschetz hyperplane section theorem. Derived functors, cohomology of sheaves, Cech cohomology, Cohomology of projective space, Serre duality, semicontinuity theorems, Zariski's main theorem and Zariski's connectedness theorem, Fulton–Lazarsfeld connectedness theorem. Applications to Brill-Noether theory.

Resource Persons	
Suresh Naik	K. N. Raghavan
D. S. Nagaraj	TEV Balaji
A. J. Parameswaran	V. Balaji
Vivek Mallik	

Eligibility for Participation: The school will admit 30 students in their first and second years of Ph.D. programme, and a few young university lecturers and college teachers. Students who have attended AFS-I/II before will be given preference to attend this school.

Financial Support: Selected participants will be paid III-AC return train fare from their place of work/home town to the venue by shortest route and provided with accommodation and local hospitality.

How to Apply: The syllabus, application form and other information about the programme is available on the website:

`http://www.bprim.org/atm`

Applications may also be made on plain paper, giving the following information:

Name, Date of Birth, Age, Gender, Institute/Department, Areas of interest, Address for correspondence, email address, City, State, Pincode, Academic Record: B.Sc./M.Sc. with names of the Institutes. These should be attested by Head/ Principal of the institute.

Completed Application Forms Should Reach:

A. J. Parameswaran & V. Balaji
KSOM, Kunnamangalam (PO), Kozhikode,
9446429164 (Mobile), 0495 2809001(Office),
0495 2809000 (KSOM Office)
E-mail: `param@math.tifr.res.in`,
`balaji@cmi.ac.in`

by **Friday**, **14th May**, 2010. List of selected candidates will be posted on the website of ATM Schools on **Friday 21th May, 2010**.

NBHM Committee for the ATM Programme	
Prof. S. A. Katre	Pune U., Pune
Prof. S. Kesavan	IMSc, Chennai
Prof. Shobha Madan	IIT Kanpur
Prof. N. Nitsure	TIFR, Mumbai
Prof. J. K. Verma (*Convener*)	IIT Bombay

Advanced Training in Mathematics Schools

Supported by
National Board for Higher Mathematics

Advanced Training School for Lecturers in Linear Algebra

5–17 July, 2010

Venue: Department of Mathematics, IIT Guwahati, Guwahati, Assam

Conveners: Vinay Wagh & Anupam Saikia

ATM School for Lecturers in Linear Algebra

Brief Description of the School: The proposed ATML is intended primarily for the college teachers in the North-East India. Starting with the basic concepts from Linear Algebra, more advanced topics will be introduced in a gradual manner. Emphasis will be on examples and exercises so that the participants develop a feeling for the subject. The tutorials in the afternoon session will complement the topics covered in the morning session, so that the key ideas are reinforced. Progress of each individual participant will be assessed by the tutors. Discussions and questions will be encouraged. The idea is to have more interaction between the resource persons and the participants.

Financial Support: Selected participants will be paid III-AC return train fare from their place of work/home town to the venue by the shortest route and provided with accommodation and local hospitality.

Eligibility for Participation: Applications are invited from lecturers in mathematics who have passed NET/SET or equivalent examination and who are teaching at a college/ university. Students doing M.Phil. may also be considered for the school. Teachers below the age of 30 will be given preference.

How to Apply: The syllabus, applications form and other information about the programme is available on the website:

`http://www.bprim.org/atm`

Application may also be made on plain paper, giving the following information:

Name, Date of Birth, Age, Gender, Institute/Department, Areas of Interest, Address for Correspondence, Email Address, City, State, Pin-code, Academic Record: B.Sc./M.Sc. with Names of the Institutes and Additional Information (if any). These should be attested by Head/Principal of the institute.

Completed Application Forms Should Reach:

Vinay Wagh & Anupam Saikia
Coordinators, ATML
Department of Mathematics
IIT Guwahati
Guwahati, Assam
Pin: 781 039

Phone:
(O): 0361 2582623 (V. Wagh), 0361 2582616 (A. Saikia)
(R): 0361 2584623 (V. Wagh), 0361 2584616 (A. Saikia)
(M): 09957 563 026 (V. Wagh), 09864 052 112 (A. Saikia)

Email: `vinay.wagh@iitg.ernet.in`,
`a.saikia@iitg.ernet.in`

by **Monday**, **31st May, 2010**. List of selected candidates will be posted on the websites of ATM Schools on **Saturday, 5th June, 2010**.

Resource Persons	
S. M. Garge	IIT Bombay
K. D. Joshi	IIT Indore
D. P. Patil	IISc, Bangalore
P. Saikia	NEHU, Shillong
Vinay Wagh	IIT Guwahati
Unity of Mathematics Lectures	
Ravi Rao, TIFR, Mumbai	

Advanced Training in Mathematics Schools

Supported by

National Board for Higher Mathematics

Workshop on Riemannian Geometry

19–30 July, 2010

Venue: TIFR CAM, Bangalore

Conveners: C. S. Aravinda & Harish Seshadri

Workshop on Riemannian Geometry

The aim of this advanced workshop is to discuss basic Riemannian geometry together with important examples and focus on certain techniques which have proved very useful over the years. We will also discuss some well-known structure theorems which reflect topological consequences of certain signs on curvatures.

Eligibility for Participation: The workshop will admit 30 students in their second and third years of Ph.D. programme, postdoctoral fellows and some college/university teachers.

Financial Support: Selected participants will be paid III-AC return train fare from their place of work/home town to the venue by shortest route and provided with accommodation and local hospitality.

How to Apply: The syllabus, application form and other information about the programme are available on the website:

`http://www.bprim.org/atm`

Applications may also be made on plain paper, giving the following information:

Name, Date of Birth, Age, Gender, Institute/Department, Areas of interest, List of Ph.D. courses (including reading courses) completed, ATM Schools attended, Any other workshop in mathematics attended, Address for correspondence, email address, City, State, Pin Code, Academic Record: B.Sc./M.Sc. with names of the institutes. These should be attested by Head/Principal of the institute.

Completed Applications May be Sent by Mail or a Scanned Copy May be Sent by E-mail to:

Prof. C. S. Aravinda
Convener, ATMWin Riemannian Geometry

TIFR Centre for Applicable mathematics,
P. O. Box 6503, Sharada Nagara, G. K. V. K. Post,
Bangalore 560 065.
E-mail: aravinda@math.tifrbng.res.in
harish@math.iisc.ernet.in
Phone: C. S. Aravinda-(080) 6695 3739 (Office)
Harish Seshadri-(080) 2293 2709 (Office)
Fax: (080) 6695 3799

by **Wednesday**, **19 May 2010**. List of selected candidates will be posted on the ATM School website on **Tuesday, 25 May 2010**.

Resource Persons	
C. S. Aravinda	Siddhartha Gadgil
S. Kumaresan	Anandateertha Mangasuli
G. Santhanam	Harish Seshadri
K. Shankar	Kaushal Verma
Guest Speakers	
T. N. Venkataramana, Gautam Bharali	

Ramanujan Institute for Advanced Study in Mathematics

University of Madras
Chepauk, Chennai 600 005

Third Summer Training Program in Mathematics – 2010

It is a pleasure to inform that Ramanujan Institute for Advanced Study in Mathematics, University of Madras, Chennai 600 005 is organizing **Third Summer Training Program in Mathematics** during **May 31 – June 19, 2010.** The Program is promoted and funded by Science City, Department of Higher Education, Government of Tamil Nadu. The primary objective of the program is to attract talented Post graduate students in Mathematics by exposing them to the power and beauty of mathematics. This program will also give hands on experience in doing mathematics and mathematical thinking. The program would consist of lectures in some basic topics by experts from leading Institutions and provide some glimpses of selected topics of current research interest.

All selected candidates will be provided with working lunch. Accommodation will be arranged for outstation candidates, and they will be paid Sleeper Class as per the rules.

Eligibility: First year M.Sc., Completed.

Application for participation in Third Summer Training Program in Mathematics, 2010

(i) Name :

(ii) Address for communication, E-mail and phone number :

(iii) Complete address of the college in which the candidate is studying :

(iv) A photo copy of the B.Sc and I semester M.Sc. mark sheets :

(v) A letter of recommendation from a mathematics teacher of the candidate :

(vi) Accommodation is required or not :

Filled in Application may be sent via E-mail: mpitchaimani@yahoo.com or Dr. M. Pitchaimani, Ramanujan Institute for Advanced Study in Mathematics, University of Madras, Chepauk, Chennai 600 005 on or before **May 12, 2010.**

The readers may download the Mathematics Newsletter from the RMS website at
www.ramanujanmathsociety.org